JME 3700
April 3, 2017

Homework #10

1. (10 pts) A pump increases the pressure in a certain piping system. The pressure rise across this pump, ΔP, depends upon the fluid density ρ, the volume flow rate Q, and the impeller diameter and rotation rate, D and ω, respectively. Derive a set of dimensionless parameters for this configuration, i.e. where dimensionless pressure rise is a function of other dimensionless number(s).

2. (10 pts) Under certain circumstances, the statement of conservation of momentum simplifies to *Prandtl's boundary layer equation*

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} ,$$

where x and y are space coordinates, $u(x, y)$ and $v(x, y)$ are velocity components, and ν is kinematic viscosity. If L and u_∞ are appropriate length and velocity scales, respectively, e.g. we can define dimensionless variables such as $x^* = x/L$ and $u^* = u/u_\infty$, re–derive this equation in dimensionless form, showing that the Reynolds number, $Re = u_\infty L/\nu$, arises as the natural parameter.

3. (10 pts) The equation in Question 2 can be generalized somewhat to describe the flow arising from buoyancy–induced convection by adding a term, as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\,\beta\,T ,$$

where $T = T(x, y)$ is a temperature distribution (units of K), g is gravitational acceleration, and β is the thermal expansion coefficient of the fluid (in units of K^{-1}). Show by similar operations as used in Question 2 and by the rules of "dimensionless algebra", that if temperature is non–dimensionalized as $T^* = T/\Delta T_R$, where ΔT_R is a reference temperature difference in units of K, a new dimensionless group called the Grashof number (the relative importance of buoyancy effects to viscous effects) appears, as defined by

$$Gr = \frac{g\,\beta\,\Delta T_R\,L^3}{\nu^2} .$$

4. (10 pts) Your engineering group is charged with characterizing the performance of a new aircraft carrier hull design. The design is only a proposal and will not actually be built unless it will give an appreciable increase in performance over current configurations. Assume the flow is governed primarily by the effects described by the Reynolds and the Froude numbers,

$$Re = \frac{U_s\,L_s}{\nu} \qquad \text{and} \qquad Fr = \frac{U_s}{\sqrt{g\,L_s}} ,$$

respectively. Here, U_s is the ship's cruising speed and L_s is its hull length. If the proposed design specifies $U_s = 10\ m/s$ and $L_s = 350\ m$, comment on whether you could study the performance experimentally using a 1/100 scale model ship towed in a water channel. Assume $\nu = 1 \times 10^{-6}\ m^2/s$ for water.

5. (10 pts) A cylinder of diameter D bobs up and down in a pool (Fig. 1). The bobbing frequency ω is assumed to be a function of D, as well as the cylinder's mass, m, and the specific weight of the liquid, γ, i.e. $\omega = \omega\,(D, m, \gamma)$. Cast this relationship in terms of relevant dimensionless variables. Also, determine whether the bobbing frequency increases or decreases if the mass of the cylinder is decreased. *Hint.* If a physical system has only a single dimensionless variable, that variable is a constant.

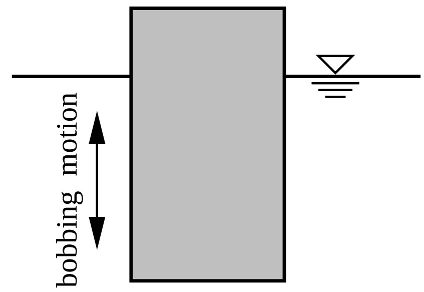

Fig. 1: *Bobbing cylinder.*