JME 3700
January 24, 2017

Homework #1 Solutions

1. (10 pts) Assuming air density is a *constant* $1.23\,kg/m^3$, calculate the change of air pressure from the top of a mountain to its base if the elevation is 3000 meters. Give your answer in kPa.

   Solution: Calculate directly from the hydrostatic equation as $\Delta P = \rho\, g\, \Delta h = 36\,kPa$

2. (10 pts) Using the concept of infinitesimals, show that $dy/dx = e^x$ is the first derivative of $y = e^x$, i.e. that this function is its own derivative, based on the observation that "higher order terms" are negligible. *Hint:* Use the fact that the exponential of any argument β is the series expansion

$$e^{\beta} = 1 + \beta + \frac{\beta^2}{2!} + \frac{\beta^3}{3!} + \frac{\beta^4}{4!} + \frac{\beta^5}{5!} + \cdots$$

   Solution: We can think of an (x, y) point on the curve $y = e^x$ and another point very close by, whose coordinates are $(x + dx, y + dy)$. Since the second point is still on this curve, it must also obey $y + dy = e^{x+dx}$. Subtracting y from both sides and making the substitution $y = e^x$, we find

$$dy = e^{x+dx} - e^x = e^x\, e^{dx} - e^x = e^x \left(e^{dx} - 1\right).$$

   Now, substitute the series expansion for e^{dx} to obtain

$$dy = e^x \left(1 + dx + \frac{(dx)^2}{2!} + \frac{(dx)^3}{3!} + \frac{(dx)^4}{4!} + \frac{(dx)^5}{5!} + \cdots - 1\right)$$

   from which we see that unity cancels and all terms of "higher order smallness" as compared to dx, e.g. those involving $(dx)^2$, $(dx)^3$, etc. can be dropped. What remains is

$$dy = e^x\, dx \qquad \text{which implies} \qquad \frac{dy}{dx} = e^x\,.$$

3. (10 pts) Later we will study the concept of drag force on a vehicle, F_D, for which we'll develop a general expression of the form

$$F_D = \frac{\rho\, u^2 A\, C_D}{2}\,,$$

   where ρ is fluid density, u and A are characteristic vehicular velocity and area, respectively, and C_D is a shape–dependent parameter. Infer the physical units of C_D.

   Solution: The left–hand side of the equation is a force, for which the units are Newtons, or more basically in the SI system

$$\frac{kg \cdot m}{sec^2}$$

   while for the right–hand side, we might write an expression like

$$\underbrace{\frac{kg}{m^3}}_{\rho} \cdot \underbrace{\frac{m^2}{sec^2}}_{u^2} \cdot \underbrace{m^2}_{A} \cdot \underbrace{kg^a\, m^b\, sec^c}_{C_D}\,,$$

where a, b, and c are to be determined such that units match–up properly. However, upon inspection, it apears that the product $\rho\, u^2 A$ already has units of force, leaving us to conclude that C_D is unitless, i.e. $a = b = c = 0$.

4. (10 pts) Fig. 1 shows a conceptually straightforward way of measuring the viscosity, μ, of a Newtonian fluid. Two parallel plates are separated by $h = 0.001\, m$. The bottom one is fixed, while the top is pulled in–plane. By making some measurements, you are able to infer a constant applied shear stress of $\tau = 1\ N/m^2$ in this experiment. If the resulting velocity profile is linear (as depicted in the figure), with the fluid's speeds at $y = 0$ and $y = h$ being $u = 0$ and $u = u_0 = 1\ m/s$, respectively, determine an expression for μ in terms of τ, u_0, and h and then substitute values to obtain a numerical estimate of μ.

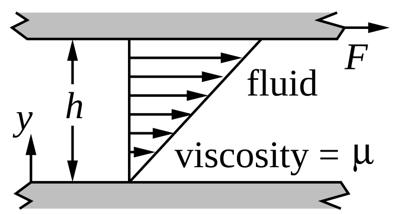

Figure 1: *Simple viscometer.*

Solution: Given that the profile is linear and that the end–conditions are

$$u\Big|_{y=0} = 0 \quad \text{and} \quad u\Big|_{y=h} = u_0\,,$$

the velocity must be given by the expression

$$u = \frac{u_0}{h}\, y\,, \quad \text{whereby} \quad \frac{du}{dy} = \frac{u_0}{h}\,.$$

Using Newton's Law of Viscosity, we find

$$\tau = \mu\, \frac{du}{dy} = \mu\, \frac{u_0}{h}\,, \quad \text{whereby} \quad \mu = \frac{\tau\, h}{u_0}\,.$$

Substituting values, we find

$$\mu = \frac{\tau\, h}{u_0} = \frac{1 \cdot 0.001}{1}\, \frac{\frac{N}{m^2} \cdot m}{\frac{m}{s}} = 0.001\, \frac{N \cdot s}{m^2}\,.$$

5. (10 pts) The simple viscometer in Question 4 can be realized physically by filling the small gap between 2 concentric cylinders with the fluid, holding the inner cylinder fixed, and rotating the outer cylinder (partial cut–away shown in Fig. 2). If the outer cylinder surface is located at a radius of $R = 0.1\ m$ and the cylinders have a length of $L = 0.5\ m$, determine the resisting torque, T, that would be realized for the parameters in Question 4. Neglect any edge effects.

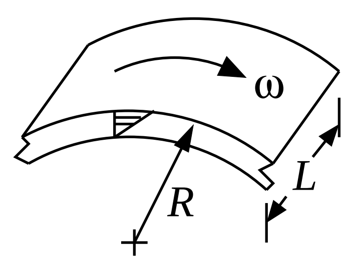

Figure 2: *Cylinders.*

Solution: Question 4 states that the shear stress is constant, so the associated force is simply $F = \tau \cdot A$, where A is the surface area of the rotating cylinder, i.e.

$$A = 2\,\pi\, R\, L = 2 \cdot \pi \cdot 0.1 \cdot 0.5 \approx 0.31416\, m^2\,.$$

$$F = \tau \cdot A = 1 \cdot 0.31416 = 0.31416\, N\,.$$

Since this acts at right angles at a uniform radius of $R = 0.1\, m$, the torque is

$$T \;=\; F \cdot R \;=\; 0.31416 \cdot 0.1 \;\approx\; 0.0314\, N \cdot m\,.$$