JME 3700
January 28, 2017

Homework #2 Solutions

1. (10 pts) Two tanks are connected, as shown in Fig. 1. The left one contains oil of density ρ_o under a certain air pressure P_M, as given by meter M mounted on its dome. The right one contains the same kind of oil, but is open to the atmosphere. A manometer having a fluid of density ρ_m connects the tanks. Given the measured displacements h_1, h_2, and h_3, determine the P_M that the gauge should display. Points **A** and **B** in Fig. 1 denote the interfaces between the oil and manometer fluid.

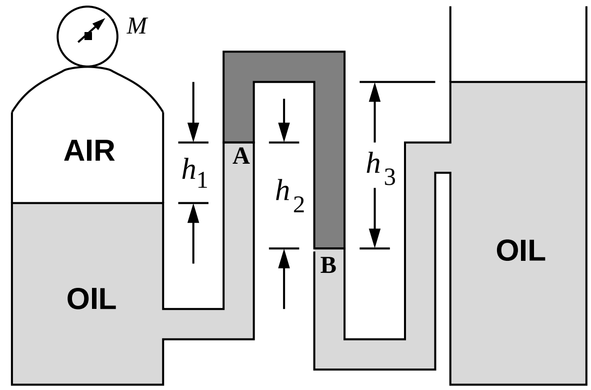

Figure 1: *Dual tank configuration.*

Solution: Let us start at the liquid surface of the open tank on the right, assuming zero gauge pressure. We proceed to walk through the system, keeping track of pressure changes, i.e. at point **B**, the pressure is $P_{\mathbf{B}} = \rho_o\, g\, h_3$. Note, we have jumped across the right–side $\bigcup$–section at constant pressure. At point **A**, the pressure is

$$P_{\mathbf{A}} = P_{\mathbf{B}} - \rho_m\, g\, h_2 = \rho_o\, g\, h_3 - \rho_m\, g\, h_2 \ ,$$

where we have now jumped across the left–side $\bigcap$–section at constant pressure. The pressure at the oil/air surface in the left tank is equal to P_M because we neglect hydrostatic contributions of the air, since its density is small compared to those of the liquids. Therefore, at the oil/air surface, we have

$$P_M = P_{\mathbf{A}} + \rho_o\, g\, h_1 = \rho_o\, g\, h_3 - \rho_m\, g\, h_2 + \rho_o\, g\, h_1 = \rho_o\, g\, (h_1 + h_3) - \rho_m\, g\, h_2 \ .$$

2. (10 pts) A rectangular gate having a width w "into the paper" separates 2 different liquid pools and is pinned at its base with a frictionless hinge (Fig. 2). The left liquid has a density ρ and a depth y_1, while for the right the respective quantities are $4\,\rho/3$ and y_2. Both y_1 and y_2 are smaller than the height of the gate. Determine the relationship between y_1 and y_2 such that the hydrostatic loads on both sides are such that the gate remains in static equilibrium in the perpendicular position suggested in the figure. That is, determine C in the equation $y_1 = C\, y_2$.

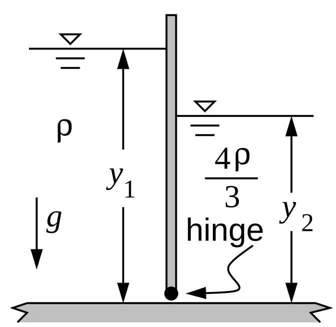

Figure 2: *Swivel gate.*

Solution: The centers of pressure on the left and right are at $y_1/2$ and $y_2/2$, respectively, meaning the respective magnitudes of the hydrostatic loads are

$$|F_1| \;=\; \rho\, g\, \frac{y_1}{2}\, y_1\, w \;=\; \frac{\rho\, g\, y_1^2\, w}{2}$$

$$|F_2| \;=\; \frac{4\,\rho}{3}\,g\,\frac{y_2}{2}\,y_2\,w \;=\; \frac{2\,\rho\,g\,y_2^2\,w}{3}\,.$$

Also, the lines of action of these forces are one–third from the bottom of the gate on their respective sides, i.e. $y_1/3$ and $y_2/3$ above the hinge. According to the requirement of static equilibrium, free–body considerations indicate the hinge reaction forces and the weight of the gate act *through* the hinge, thus imparting no moment *about* the hinge. Only the hydrostatic loads result in moments about the hinge and equilibrium requires that these moments sum to zero, i.e. $|F_1| \cdot y_1/3 = |F_2| \cdot y_2/3$. Consequently,

$$\frac{\rho\,g\,y_1^2\,w}{2} \cdot \frac{y_1}{3} \;=\; \frac{2\,\rho\,g\,y_2^2\,w}{3} \cdot \frac{y_2}{3}$$

$$\frac{y_1^3}{6} \;=\; \frac{2\,y_2^3}{9}$$

$$y_1^3 \;=\; \frac{4}{3}\,y_2^3 \qquad y_1 \;=\; \left(\frac{4}{3}\right)^{1/3} y_2 \;\approx\; 1.1\,y_2\,.$$

3. (10 pts) A "reservoir"–type manometer (Fig. 3) is to be calibrated for use with a fluid of specific gravity S_G. The vertical tube diameter (at section 2) is D_2, while the reservoir diameter (at section 1) is D_1, where $D_1 > D_2$. Calculate the deflection distance h_2 on the vertical tube per h_w of applied water pressure difference. That is, find the ratio of h_2 to h_w, the latter being what the *analogous* reading would be in a standard (simple) water U–tube manometer. (The dashed line is the equilibrium state of the instrument when no pressure is applied.) Assume that some amount of manometer fluid always remains with the reservoir.

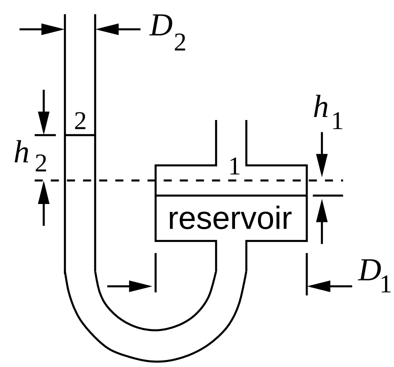

Figure 3: *Reservoir manometer.*

Solution: Assume a pressure difference between sections 1 & 2 displaces the fluid upward in the vertical tube by an amount h_2 and the fluid in the reservoir downward by an amount h_1. From the hydrostatic equation, we can write $P_2 \,+\, S_G\,\rho_w\,g\,(h_1 \,+\, h_2) \,=\, P_1$, so that

$$P_1 \,-\, P_2 \;=\; S_G\,\rho_w\,g\,(h_1 \,+\, h_2)\;,$$

where ρ_w is the density of water. The volume of manometer fluid is constant, i.e. the amount displaced in the reservoir equals the amount that rises in the tube:

$$\frac{\pi\,D_2^2\,h_2}{4} \;=\; \frac{\pi\,D_1^2\,h_1}{4}\,,$$

so that $h_1 = h_2\,(D_2/D_1)^2$. The pressure difference measured is equal to that measured in a simple U–tube manometer having an equivalent water column of height h_w. In terms of water, we can write $P_1 - P_2 = \rho_w\,g\,h_w$. Combining these three equations, we find

$$\rho_w\,g\,h_w \;=\; S_G\,\rho_w\,g\,\left(h_2\left[\frac{D_2}{D_1}\right]^2 \,+\, h_2\right)$$

$$
\begin{aligned}
h_w &= S_G\, h_2 \left(\left[\frac{D_2}{D_1} \right]^2 + 1 \right) \\
\frac{h_2}{h_w} &= \frac{1}{S_G \left(D_2^2 / D_1^2 + 1 \right)} .
\end{aligned}
$$

4. (10 pts) A cube of edge length d floats precisely half submerged in a pool of water (Fig. 4). If the cube and water have densities of ρ_c and ρ, respectively, find the relationship of these two densities to one another.

Figure 4: *Floating cube.*

Solution: The weight of the cube is $W = \rho_c\, g\, d^3$, while the buoyancy force, equal to the weight of the water displaced, is $F_B = \rho\, g\, d^3/2$, since the volume displaced is exactly half the cube's volume. Since the system is static, $W = F_B$, whereby

$$
\rho_c\, g\, d^3 = \frac{\rho\, g\, d^3}{2} ,
$$

from which we readily find $\rho_c = \rho/2$, i.e. the cube material is half the density of water.

5. (10 pts) A simple control system allows liquid to be dumped when its level rises sufficiently (Fig. 5). Specifically, a gate is hinged at point A and will open by rotating clockwise if D becomes large enough. Assuming the gate has a depth w into the paper, at what D will the gate just start to open if the liquid has a density ρ? Assume the vertical section is tall enough, such that the liquid does not spill over the top before the gate opens. Also, assume that the friction in the hinge and weight of the gate (and any moment imparted) are all small enough in this case to be neglected.

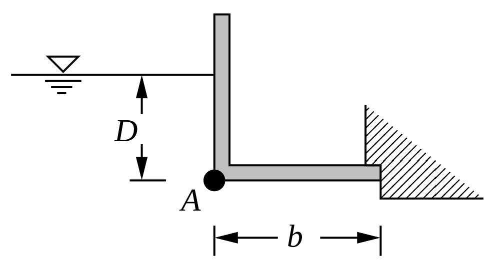

Figure 5: *Right–angle gate.*

Solution: Pressure on the horizontal surface is constant (since it is horizontal). Therefore, the upward force F_h on the horizontal part of the gate is uniform $F_h = \rho\, g\, D\, w\, b$ and it is located $b/2$ away from the hinge point. The force on the vertical section is the pressure at the centroid times the area, i.e. $F_v = 0.5\, \rho\, g\, D \times D\, w$ and it is located at a distance $D/3$ above the hinge point. The gate will be free to swing open once the moments about the hinge point vanish, i.e. when

$$
\rho\, g\, D\, w\, b \times \frac{b}{2} = \frac{\rho\, g\, D^2\, w}{2} \times \frac{D}{3} .
$$

Solving, we find $D = \sqrt{3}\, b$.