JME 3700
March 15, 2017

Homework #8

1. (10 pts) A certain flow is described by the potential function $\phi = x^3y - xy^3 + x - y + 6$. Determine the stream function ψ to within some additive constant C for this flow.

2. (10 pts) It has been claimed that a necessary condition for describing fluid motion using a potential function is that the vorticity, or rotation, must be zero. Determine whether the motion described in Question 1 is consistent with this claim.

3. (10 pts) A certain incompressible flow is described by the stream function $\psi = 4x^2y^2 - y^4$. Find the equation(s) of the streamline(s) passing through the origin.

4. (10 pts) A particular two–dimensional velocity field is given by

$$\mathbf{V} = u_0 \sin\left(\frac{x}{L}\right) \hat{i} - \frac{u_0\, y}{L} \cos\left(\frac{x}{L}\right) \hat{j}\,,$$

   where u_0 and L are constants having units of speed (m/s) and length (m), respectively. Determine the volumetric dilation rate and give a physical interpretation of the result. Also, determine the vorticity and state whether the flow is irrotational or not.

5. (10 pts) You are trying to determine the velocity field $\mathbf{V} = u\,\hat{i} + v\,\hat{j} + w\,\hat{k}$ for a particular 3–dimensional incompressible flow. A special 2–dimensional flow measurement device indicates that two of the components are $u = x^2y + xz$ and $v = -xy^2 + yz$ and then tells you that you're on your own for the third component, w. Determine w to within an additive function $f(\,)$. Make sure to note the dependent variables of this function.