JME 3700
March 24, 2017

Homework #9

1. (10 pts) A viscous, incompressible fluid flows steadily between two infinite parallel plates separated by a distance b (Fig. 1). Motion is driven by the plates themselves, the bottom and top plates sliding in their own planes with velocities u_o and $2u_o$, respectively. There is no pressure gradient. Determine the horizontal velocity distribution if the flow is laminar and fully–developed and the fluid has density and viscosity of ρ and μ, respectively and use this result to subsequently determine the shear stress on the bottom plate.

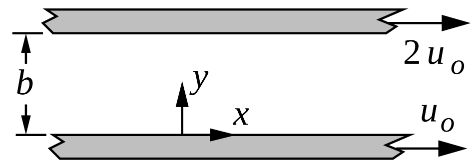

Fig. 1: *Flow between plates.*

2. (10 pts) The flow in Question 1 has no variation in the z–direction (i.e. $+z$ coming "out of the paper") and no velocity component in this direction, either. In such instances, the vorticity (which in general is a vector as is velocity) has only one component of vorticity. Determine the vorticity for the flow in Question 1 and comment on how fluid elements are "tumbling" in this flow.

3. (10 pts) A particular flow is described by the potential function $\phi = C_0\, x\, y$, where C_0 is a constant. Find the corresponding stream function, ψ to within an additive constant C_1.

4. (10 pts) Show by either integrating the following form of the Euler equations,

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} \qquad \text{and} \qquad \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial P}{\partial y} \, ,$$

or by applying the Bernoulli equation for potential flows (neglect potential energy), that the pressure distribution in Problem 3 is

$$P = - \frac{\rho\, C_0^2}{2} \left(x^2 + y^2 \right) + C_2 \, ,$$

where C_2 is an additive constant.

5. (10 pts) Consider fully–developed, steady laminar flow in a pipe of inner radius R (Fig. 2), whose axial profile is

$$u(r) = \frac{1}{4\,\mu} \frac{dP}{dx} \left(r^2 - R^2 \right) \, ,$$

where pressure gradient dP/dx and viscosity μ are both constants. Here, the x–axis and the flow direction are "into the paper". Determine the shear stress, τ, realized at the inner wall (at $r = R$) in terms of the volume flow rate, Q, the radius, R, and the fluid viscosity, μ.

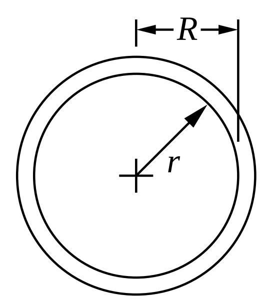

Fig. 2: *Flow in a pipe.*