JME 3700
Midterm Exam: May 3, 2017 **Name:** ____________________

Solutions

1. A car sits on a circular platform of diamater D, which is supported by a liquid of density ρ and which is free to move frictionlessly either up or down (Fig. 1). The tank containing the liquid has a standard manometer attached for reading pressure. The car and the platform together have a mass of M and gravitational acceleration, g, is downward, as shown. This system is at static equilibrium.

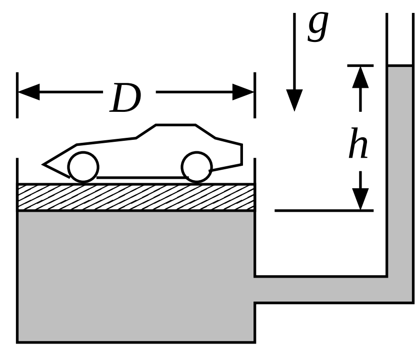

Fig. 1: *Car platform*

(a) (15 pts) Determine the pressure, P_1, within the liquid at the bottom face of the platform (i.e. just underneath the car) as a function of the weight it is supporting and assuming it is uniformly distributed over the platform's surface.

Solution: If the pressure is uniform, then $P_1 A = W$, where $A = \pi\, D^2/4$ is the platform's area and $W = M\, g$ the total supported weight. Thus,

$$P_1 \;=\; \frac{M\,g}{\pi\,D^2/4} \;=\; \frac{4\,M\,g}{\pi\,D^2}\;.$$

(b) (15 pts) Determine the corresponding reading, h, of the manometer for pressure P_1. Give your final answer as a numeric value in units of meters if $\rho = 1000\,kg/m^3$, $D = 4\,m$, and $M = 2000\,kg$.

Solution: Starting at P_1 and jumping across to the same elevation at constant pressure, we see that the open–end of the manometer is at zero pressure, so that $P_1 \,-\, \rho\,g\,h \,=\, 0$, whereby

$$h \;=\; \frac{P_1}{\rho\,g} \;=\; \frac{4\,M\,g}{\pi\,D^2\,\rho\,g} \;=\; \frac{4\,M}{\pi\,D^2\,\rho} \;=\; \frac{4\cdot 2000}{\pi\cdot 4^2\cdot 1000} \;\approx\; 0.159\,m\,.$$

2. A horizontal stream of liquid having density ρ, cross–sectional area A, and speed u_0 flows steadily in the $+x$ direction and is deflected at an angle θ by a plank attached to a bob of weight W and hinged with a frictionless pin at point α (Fig. 2). The bob's mass center is a distance h from the pin, while the reaction force of the fluid deflection can be taken to be at a point β, which is a distance $3\,h$ from the pin. Assume uniform velocity profile, cross–section A is maintained constant, and changes in potential energy are negligible.

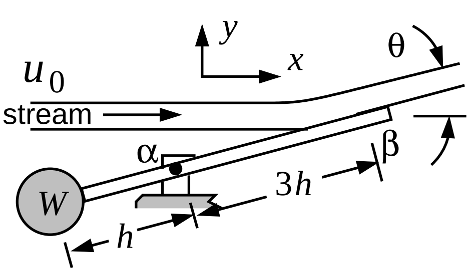

Fig. 2: *Deflecting stream*

(a) (15 pts) Determine the vertical component of the reaction force, R_y, experienced by the plank via its deflection of the steady–state liquid stream.

Solution: We can start with the steady version of the equation for the vertical component of momentum. Assuming R_y to act the in the $+y$ direction, we have

$$R_y \;=\; \iint v\,\rho\,\mathbf{V}\cdot\hat{n}\,dA\,,$$

where v is the vertical component of momentum (which is transported). Assuming a control volume around the stream and noting that no vertical momentum is transported in the horizontal part (the "inlet"), but that there *is* a component in the deflected part (the "outlet"), we can write

$$R_y \;=\; \iint\limits_{deflected} v\,\rho\,\mathbf{V}\cdot\hat{n}\,dA \;=\; \Big(u_0\,\sin\theta\Big)\,\rho\,\Big(+u_0\Big)\,A \;=\; \rho\,u_0^2\,A\,\sin\theta\,.$$

Note that we have skipped actual integration because the velocity profile is constant and that we determined the "+" sign from the dot product.

(b) (15 pts) Let conditions be such that the deflection angle θ is extremely small, i.e. the plank is *almost* horizontal. (Under these conditions $\sin\theta\sim\theta$ and $\cos\theta\sim 1$.) Neglecting the weight of the plank itself, determine θ as a function of the bob's weight, W and the parameters associated with the flowing stream.

Solution: Since the plank is almost horizontal, we can sum moments about the frictionless pin assuming essentially vertical loading, from which we find $h\cdot W = 3\,h\cdot R_y$. Substituting the small–angle approximation of R_y, we see

$$h\,W \;=\; 3\,h\,\rho\,u_0^2\,A\,\theta\,,$$

from which we can solve

$$\theta \;=\; \frac{W}{3\,\rho\,u_0^2\,A}\,.$$

3. Water of density ρ flows inviscidly and steadily in a horizontal pipe of diameter D_1 with a uniform velocity V_1 and pressure P_1 (Fig. 3). The latter is measured by a static port connected to a manometer. The flow meets a smoothly and gradually necked–down constriction of diameter D_2, where the uniform velocity is V_2. A pitot tube situated precisely in the throat measures the stagnation pressure, P_2. The manometer is filled with a fluid of specific gravity S_g and shows a displacement a. Gravity, g, points in the negative y–direction.

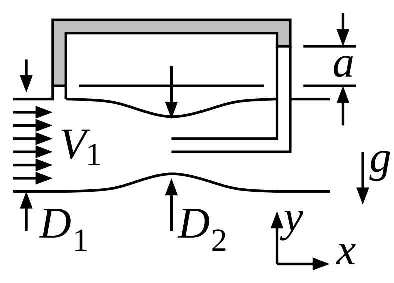

Fig. 3: *Velocity measurement*

(a) (15 pts) Calculate the throat velocity, V_2, in terms of the (still unknown) V_1 and the diameters.

Solution: According to conservation of mass, the volumetric flow rate must be constant, whereby

$$\frac{\pi\, D_1^2}{4}\, V_1 \;=\; \frac{\pi\, D_2^2}{4}\, V_2 \qquad \rightarrow \qquad V_2 \;=\; \frac{D_1^2\, V_1}{D_2^2}\,.$$

(b) (15 pts) Determine a more informative result for V_2 by solving for V_1 and substituting. Your solution for latter should be a function of g, a, and S_g. *Hint:* Use the fact that stagnation conditions exist at the pitot tube entrance.

Solution: The Bernoulli equation can be written between points 1 (upstream) and 2 (pitot tube entrance) as

$$P_1 \;+\; \frac{\rho\, V_1^2}{2} \;+\; \rho\, g\, z_1 \;=\; P_2 \;+\; \frac{\rho\, V_2^2}{2} \;+\; \rho\, g\, z_2\,,$$

but the horizontal orientation means the potential energy terms vanish and stagnation conditions at the pitot tube mean that V_2 vanishes. Simplifying, we find

$$V_1 \;=\; \sqrt{\frac{2\,(P_2 - P_1)}{\rho}}\,,$$

however, we also observe from the manometer reading that

$$P_1 \;-\; S_g\, \rho\, g\, a \;+\; \rho\, g\, a \;=\; P_2 \qquad \text{whereby} \qquad P_2 \;-\; P_1 \;=\; \rho\, g\, a\,(1 - S_g)\,.$$

Finally, we get

$$V_1 \;=\; \sqrt{2\, g\, a\,(1 - S_g)} \qquad \text{so that} \qquad V_2 \;=\; \frac{D_1^2\, \sqrt{2\, g\, a\,(1 - S_g)}}{D_2^2}\,.$$

(c) (10 pts) Determine the nature of S_g, i.e. is it greater or less than 1? (In other words, is the manometer fluid more dense than water, or less?) You can base your answer on either a mathematical or a physical argument.

Solution: Either of the following arguments is acceptable.

Mathematical: In the expression for V_1, the term $1 - S_g$ appears under the square root. Since both $g > 0$ and $a > 0$, it must be the case that $S_g < 1$, whereby $1 - S_g$ is also greater than 0, such that $\sqrt{2\, g\, a\,(1 - S_g)}$ is a real result.

Physical: If S_g were greater than 1, then the manometer fluid would be "top heavy" and would tend to fall out of the manometer if disturbed by transients or at rest. Consequently, $S_g < 1$.