JME 3710 / ME 3710
Final Exam: December 19, 2016 **Name:** ____________________

Solutions

- Questions having bold–faced point values in the solution indicate the basis of how partial–credit was tallied.

1. Two spherical objects in outer space pass within $w = 600\,m$ of one another (Fig. 1, left). The smaller object has a total surface area of $A_1 = 10\,m^2$, a temperature of $T_1 = 1000\,K$, and a total emissivity of $\varepsilon_1 = 0.8$. The larger object, of radius $r = 200\,m$ (surface area $4\pi r^2$), is at a temperature of $T_2 = 3000\,K$ and has a total emissivity of $\varepsilon_2 = 0.2$. Space can be treated as an idealized radiant surrounding having a background temperature of $T_3 = 20\,K$. This problem, at the instant shown, can be modeled using the circuit analogy diagrammed in Fig. 1 (right, with reference arrows suggested), where E_b, J, R, and q represent appropriate idealized emissive powers, radiosities, resistances, and net rates of heat leaving/arriving at surfaces, respectively.

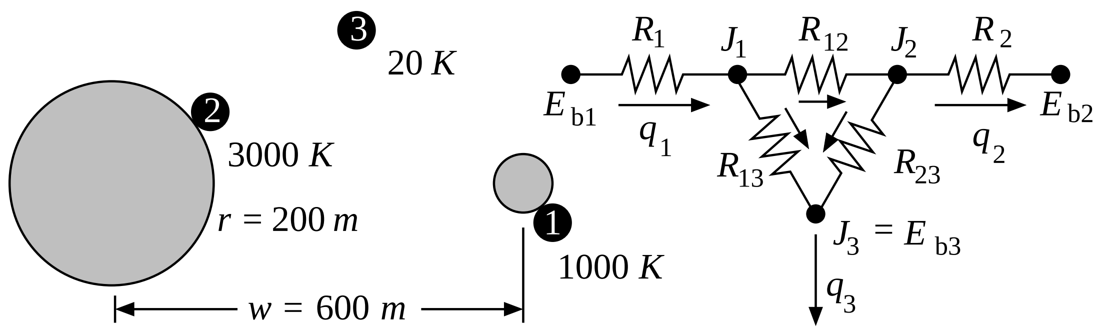

Fig. 1: *Two spheres in space (left) and corresponding circuit model (right).*

(a) (5 pts) In the radiation circuit analogy, exterior nodes can be determined directly. Calculate the values of E_{b1}, E_{b2}, and $J_3 = E_{b3}$ in units of W/m^2.

Solution: Using the Stefan–Boltzmann equation **(2 pts)**, we have the following **(1 pt each)**:

$$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8} \; \frac{W}{m^2\,K^4} \cdot 1000^4\,K^4 = 56,700 \; \frac{W}{m^2}\,,$$

$$E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-8} \; \frac{W}{m^2\,K^4} \cdot 3000^4\,K^4 = 4,592,700 \; \frac{W}{m^2}\,,$$

$$J_3 = E_{b3} = 5.67 \times 10^{-8} \; \frac{W}{m^2\,K^4} \cdot 20^4\,K^4 \approx 0.00907 \; \frac{W}{m^2}\,.$$

(b) (5 pts) Calculate values for the surface resistors R_1 and R_2 in unit of m^{-2}.

Solution: "Surface" resistors are calculated according to the general formula $(1-\varepsilon)\,(\varepsilon A)^{-1}$

(3 pts), whereby we find **(1 pt each)**:

$$R_1 \;=\; \frac{1 \,-\, \varepsilon_1}{\varepsilon_1\, A_1} \;=\; \frac{1 \,-\, 0.8}{0.8 \cdot 10} \;=\; 0.025\, m^{-2}\,,$$

$$R_2 \;=\; \frac{1 \,-\, \varepsilon_2}{\varepsilon_2\, A_2} \;=\; \frac{1 \,-\, 0.2}{0.2 \cdot 4 \cdot \pi \cdot 200^2}$$

$$=\; \frac{0.8}{0.2 \cdot 502,655} \;=\; 7.96 \times 10^{-6} \;\approx\; 8 \times 10^{-6}\, m^{-2}\,.$$

(c) (15 pts) Analysis indicates that, to a very good approximation, the view factor F_{12} is given by the equation

$$F_{12} \;=\; \frac{1 \,-\, \sqrt{1 \,-\, (r/w)^2}}{2}\,.$$

Using this, along with any other appropriate observations/information, determine the values of the 3 shape–factor resistors, R_{12}, R_{13}, and R_{23} in units of m^{-2}.

Solution: Given $r = 200$ and $w = 600$, view factor F_{12} is calculated as **(2 pts)**:

$$F_{12} \;=\; \frac{1 \,-\, \sqrt{1 \,-\, (200/600)^2}}{2} \;\approx\; 0.0286\,,$$

whereby **(2 pts)**:

$$R_{12} \;=\; \frac{1}{A_1\, F_{12}} \;=\; \frac{1}{10 \cdot 0.0286} \;\approx\; 3.497\, m^{-2}\,.$$

Given **(1 pt)** $F_{11} = F_{22} = 0$ because the spheres are convex, we find **(2 pts)**:

$$F_{13} \;=\; 1 \,-\, F_{11} \,-\, F_{12} \;=\; 1 \,-\, 0 \,-\, 0.0286 \;=\; 0.9714\,,$$

whereby **(2 pts)**:

$$R_{13} \;=\; \frac{1}{A_1\, F_{13}} \;=\; \frac{1}{10 \cdot 0.9714} \;\approx\; 0.1029\, m^{-2}\,.$$

Given reciprocity, $A_1\, F_{12} = A_2\, F_{21}$, we find **(2 pts)**:

$$F_{21} \;=\; \frac{A_1\, F_{12}}{A_2} \;=\; \frac{10 \cdot 0.0286}{4 \cdot \pi \cdot 200^2} \;=\; \frac{0.286}{502,655} \;\approx\; 5.7 \times 10^{-7}$$

and **(2 pts)**:

$$F_{23} \;=\; 1 \,-\, F_{21} \,-\, F_{22} \;=\; 1 \,-\, 5.7 \times 10^{-7} \,-\, 0 \;=\; 0.9999994 \;\approx\; 1\,,$$

so that **(2 pts)**:

$$R_{23} \;=\; \frac{1}{A_2\, F_{23}} \;=\; \frac{1}{502,655 \cdot 0.9999994} \;=\; 1.929 \times 10^{-6} \;\approx\; 2 \times 10^{-6}\, m^{-2}\,.$$

(d) (10 pts) Measurements by sophisticated instruments estimate $J_2 = 918,540\,W/m^2$ for the larger sphere. Determine J_1.

Solution: Based on Kirchoff's Law, we sum the "currents" at node J_1, finding **(5 pts)**:

$$\begin{aligned}\frac{E_{b1} - J_1}{R_1} &= \frac{J_1 - E_{b3}}{R_{13}} + \frac{J_1 - J_2}{R_{12}} \\ \frac{56,700 - J_1}{0.025} &= \frac{J_1 - 0.00907}{0.1029} + \frac{J_1 - 918,540}{3.497}\end{aligned}$$

Solving algebraically, we find **(5 pts)** $J_1 \approx 50,614\,W/m^2$.

(e) (5 pts) Calculate the net rate of radiation heat transfer leaving sphere number 1, i.e. q_1.

Solution: The radiation heat transfer rate q_1 is simply the "current" through R_1, calculated as **(5 pts)**:

$$q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{56,700 - 50,614}{0.025} = 243,440\,W\,.$$

2. A simple tube–in–a–tube heat exchanger is configured in counterflow mode to recover heat from oil (the "hot side") by means of cold water (the "cold side"). Water has specific heat and mass flow rate of $c_c = 4170\,J/(kg \cdot K)$ and $\dot{m}_c = 0.48\,kg/s$. For the oil, $c_h = 2000\,J/(kg \cdot K)$ and $\dot{m}_h = 2\,kg/s$. The available heat transfer area is $A = 12.5\,m^2$. Inlet temperatures and convection coefficients are listed in Table 1.

Table 1: Other parameters

Parameter	Value
$T_{c,i}$	$20\,^oC$
$T_{h,i}$	$100\,^oC$
h_c	$2000\,W/\left(m^2 \cdot K\right)$
h_h	$500\,W/\left(m^2 \cdot K\right)$

(a) (5 pts) Presuming the use of the ε–NTU analysis method, identify the so–called minimum side ("min side") and the maximum *theoretical* heat transfer rate, q_{max}, in units of W.

Solution: The heat capacities are the products of mass flow rates and specific heats, which are calculated as **(2 pts)**:

$$\begin{aligned}\dot{m}_c \cdot c_c &= 0.48 \cdot 4170 = 2001.6 \\ \dot{m}_h \cdot c_h &= 2 \cdot 2000 = 4000\,,\end{aligned}$$

and, since the cold side has the smaller value, it is the "min side" **(1 pt)**. The maximum theoretical heat transfer rate is **(2 pts)**:

$$q_{max} = (\dot{m} \cdot c)_{min}\,(T_{h,i} - T_{c,i}) = 2001.6 \cdot (100 - 20) = 160,128\,W\,.$$

(b) (5 pts) Pressure is limited on both the hot and cold sides, such that the tubing is of relatively low thickness. Taken with the high conductivity of the material, temperature gradients associated with conduction can be neglected. If fouling effects are also negligible, calculate the overall convection coefficient, U, in units of $W/\left(m^2 \cdot K\right)$.

Solution: Under these circumstances, the idealized equation for U can be used **(2 pts)**, which yields the following answer when h_c and h_h are substituted **(3 pts)**:

$$U = \frac{1}{1/h_h + 1/h_c} = \frac{1}{1/500 + 1/2000} = 400 \ \frac{W}{m^2 \cdot K} \,.$$

(c) (10 pts) Given that the ε–NTU relationship for this particular heat exchanger design is known to be

$$\varepsilon = \frac{1 - e^{-NTU \cdot (1-C)}}{1 - C \cdot e^{-NTU \cdot (1-C)}} \qquad C = \frac{(\dot{m}\, c)_{min}}{(\dot{m}\, c)_{max}} \,,$$

calculate the effectiveness, ε, for the specified operating conditions.

Solution: The parameter C is readily calculated from above as **(2 pts)**:

$$C = \frac{(\dot{m}\, c)_{min}}{(\dot{m}\, c)_{max}} = \frac{2001.6}{4000} = 0.5004 \approx 0.5 \,.$$

The number of (heat) transfer units, NTU, is calculated using the basic definition **(3 pts)**, for which substitution of known values from above yields **(2 pts)**:

$$NTU = \frac{A \cdot U}{(\dot{m}\, c)_{min}} = \frac{12.5 \cdot 400}{2001.6} = 2.498 \approx 2.5 \,.$$

Using the ε–NTU relationship stated in the question, we fin **(3 pts)**:

$$\varepsilon = \frac{1 - e^{-2.498 \cdot (1 - 0.5004)}}{1 - 0.5004 \cdot e^{-2.498 \cdot (1 - 0.5004)}} = 0.8325 \approx 0.83 \,.$$

(d) (5 pts) Given the above information, the so–called *rating* problem for this heat exchanger can be solved, one aspect of which is the actual heat transfer rate, q. Calculate q in units of W.

Solution: Invoking the basic definition $\varepsilon = q/q_{max}$ **(3 pts)**, we substitute to find **(2 pts)**:

$$q = \varepsilon \cdot q_{max} = 0.8325 \cdot 160128 = 133306 \approx 133300 \, W \,.$$

(e) (5 pts) Another aspect of the rating problem is establishing outlet temperatures. Using any suitable method, calculate the cold side outlet temperature, $T_{c,o}$, in units of Celsius.

Solution: Invoking any basic relationship **(3 pts)**, e.g.

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} \qquad \text{or} \qquad q = \dot{m}_c \cdot c_c \, (T_{c,o} - T_{c,i})$$

we can substitute values to obtain **(2 pts)**:

$$0.8325 = \frac{T_{c,o} - 20}{100 - 20} \qquad \text{or} \qquad 133306 = 2001.6 \, (T_{c,o} - 20) \,,$$

from which we obtain $T_{c,o} \approx 87\,^oC$.

(f) (5 pts) The ε–NTU relationship for a specific heat exchanger configuration can simplify in various ways under certain conditions. Here, for example, if the min side and max side mass flow rates were increased and decreased, respectively, to very large degrees, we would find that the heat capacity ratio would tend to

$$C = \frac{(\dot{m}\,c)_{min}}{(\dot{m}\,c)_{max}} \to 0\,.$$

Determine the simplified form of the general ε–NTU relationship given above for this configuration when $C \to 0$.

Solution: We substitute $C = 0$ into the above equation, finding **(5 pts)**:

$$\varepsilon = \frac{1 - e^{-NTU\cdot(1-0)}}{1 - 0\cdot e^{-NTU\cdot(1-0)}} = \frac{1 - e^{-NTU}}{1} = 1 - e^{-NTU}\,.$$

(g) (5 pts) For this particular heat exchanger configuration, the ε–NTU relationship seems to break–down if the 2 flows are "impedance matched", i.e. when the $\dot{m}\cdot c$ product of the two sides are equal. Here, $C = 1$, which naive substitution suggests

$$\varepsilon = \frac{1 - e^{-NTU\cdot(1-1)}}{1 - 1\cdot e^{-NTU\cdot(1-1)}}\,,$$

and which subsequently leads to the indeterminate form 0/0 if carried through. Demonstrate that the actual simplification for $C = 1$ is $\varepsilon = NTU/(NTU + 1)$.

Solution: Indeterminate forms can be examined using L'Hospital's Rule **(3 pts)**. Taking d/dC of the numerator and denominator **(1 pt)**, we see

$$\begin{aligned}\varepsilon &= \frac{d/dC\left[1 - e^{-NTU\cdot(1-C)}\right]}{d/dC\left[1 - C\cdot e^{-NTU\cdot(1-C)}\right]}\\ &= \frac{0 - NTU\cdot e^{-NTU\cdot(1-C)}}{0 - e^{-NTU\cdot(1-C)} - NTU\cdot C\cdot e^{-NTU\cdot(1-C)}}\\ &= \frac{-e^{-NTU\cdot(1-C)}}{-e^{-NTU\cdot(1-C)}}\cdot\frac{NTU}{1 + NTU\cdot C}\,,\end{aligned}$$

from which the proposition follows directly when $C = 1$ is inserted **(1 pt)**.

3. Radiation is exchanged among 4 surfaces, described as: (1) a flat disk on the bottom, (2) a circular wall rising from the disk, (3) the surroundings, which are represented by a virtual, dashed surface, and (4) a flat square plate on top that is parallel to the disk (Fig. 2). The cylinder is open, i.e. there is no "top" on the cylinder. The square is not necessarily centered exactly over the disk and the configuration does *not* contain any basic surface–to–surface pairs that can be looked–up in a handbook (the textbook), but measurements indicate $F_{41} = 0.05$.

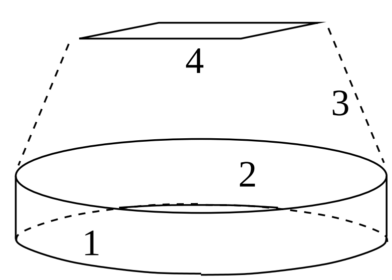

Fig. 2: *Radiation enclosure.*

(a) (5 pts) Determine how many view factors characterize this problem and write down the matrix representation of these factors.

Solution: There are $N = 4$ surfaces, which gives $N^2 = 16$ total view factors **(2 pts)**. In matrix form, these are **(3 pts)**:

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix} .$$

(b) (5 pts) Determine the value of view factor F_{44} and state the basis or reason underlying your answer.

Solution: The value is $F_{44} = 0$ **(3 pts)**, which we determine by inspection, since both flat and convex surfaces cannot "see" themselves **(2 pts)**.

(c) (5 pts) View factor F_{42} would be very difficult to calculate from first principles, i.e. by evaluating the view factor integral. However, suppose the view factor from surface 4 to a *virtual* surface V stretched across the top of the cylinder were known to be $F_{4V} = 0.08$. Use this information to find F_{42}.

Solution: Based on conservation of energy, all radiation passing through virtual surface V hits either surface 1 or surface 2, meaning **(3 pts)**:

$$\begin{aligned} q_{4\to V} &= q_{4\to 1} + q_{4\to 2} \\ J_4\, A_4\, F_{4V} &= J_4\, A_4\, F_{41} + J_4\, A_4\, F_{42} \\ F_{4V} &= F_{41} + F_{42} , \end{aligned}$$

from which we can substitute and solve **(2 pts)**:

$$F_{42} = F_{4V} - F_{41} = 0.08 - 0.05 = 0.03 .$$

(d) (5 pts) Suppose a particular proposition involved the rate at which surface 4 (the square) transfers radiation to the surroundings, for which F_{43} is the relevant view factor. Calculate F_{43} using view factor algebra.

Solution: From summation of row 4 in the view factor matrix **(3 pts)**, we have the relationship $F_{41} + F_{42} + F_{43} + F_{44} = 1$. Solving for F_{43} and substituting, we find **(2 pts)**:

$$F_{43} = 1 - F_{41} - F_{42} - F_{44} = 1 - 0.05 - 0.03 - 0 = 0.92 .$$