MEMS 5403
March 22, 2017

Homework #5

1. (10 pts) A fluid of kinematic viscosity ν is in a semi–infinite domain $y \geq 0$ bounded by a flat plate at $y = 0$. This plate oscillates in its own plane at a frequency of ω, leading to a velocity distribution similar to what is qualitatively shown in Fig. 1, i.e. where the flow is uni–directional. If the velocity $u = u(y,t)$ is defined formally by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \qquad u(y \to \infty, t) = 0$$

$$u(y, t = 0) = 0 \qquad u(y = 0, t) = U_0 \sin(\omega t)$$

and we define dimensionless variables as

$$u^* = \frac{u}{U_0} \qquad t^* = \omega t \qquad y^* = \frac{y}{\sqrt{\nu/\omega}} ,$$

show that the problem can be stated non–dimensionally in the parameter–free form

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}} \qquad u^*(y^* \to \infty, t^*) = 0$$

$$u(y^*, t^* = 0) = 0 \qquad u(y^* = 0, t^*) = \sin(t^*) .$$

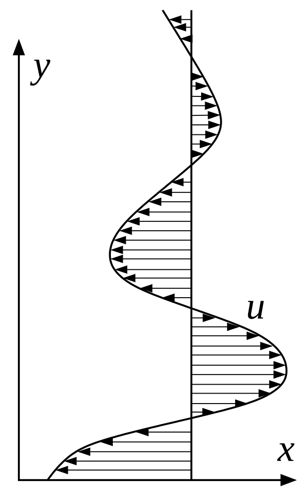

Figure 1: *Oscillating flow.*

2. (10 pts) The flow configuration in Question 1 can be represented as the superposition of a transient "start–up" component and a steady, actually *quasi–steady* persistent component. The latter property is due to the time–dependent oscillating boundary condition at $y = 0$. Show that the quasi–steady component of the solution is

$$u^* = e^{-\sqrt{2}\, y^*/2} \sin\left(t^* - \frac{\sqrt{2}\, y^*}{2} \right) .$$

Hint: Transform the problem to the complex domain, assuming the quasi–steady solution has the form $U^* = f(y^*) \cdot e^{i\,t^*}$. For instance, the oscillating boundary condition is

$$u^*(y^* = 0, t^*) = \sin(t^*) = \Im(e^{i\,t^*}) ,$$

according to Euler's formula, $e^{i\,t^*} = \cos(t^*) + i\,\sin(t^*)$, where $\Im(\,)$ indicates the imaginary component. After obtaining U^*, the physical solution can be recovered as $u^* = \Im(U^*)$.