MEMS 5403
April 7, 2017

Homework #6

1. (10 pts) The Karman–Pohlhausen integral method is a clever way to obtain good approximate solutions to the boundary layer development flow porblem for velocity distribution $u(x, y)$ in the x–direction on a flat plate. Here, the integral takes the form

$$\frac{d}{dx}\int_0^{\delta} u\big(u_\infty - u\big)\, dy = \nu \left.\frac{\partial u}{\partial y}\right|_{y=0} ,$$

where u_∞ is the freestream velocity, ν is viscosity, $\delta = \delta(x)$ is the boundary layer thicknes, and y is the direction perpendicular to the plate. Following the approach of solving for $u(x, y)$ as a function of the combined variable y/δ, a trial velocity profile in the form of a polynomial is often used. Instead, using a trial profile in the form of

$$\frac{u}{u_\infty} = a_0 + a_1 \sin\left(\frac{\pi\, y}{2\,\delta}\right) ,$$

where a_0 and a_1 are undetermined constants, solve for the boundary layer growth in the form of

$$\delta = \frac{C\, x}{\sqrt{Re_x}} ,$$

specifically determining the constant C. Compare this constant to the value of 5.0 and 4.64 from the exact and polynomial–approximate solutions, respectively.